ARTIFICIAL & NATURAL INTELLIGENCE TOULOUSE INSTITUTE

HEROIC Chair

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Assignment Problems in Cost Function Networks

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We develop an **AllDifferent** constraint propagator for solving assignment problems in Cost Function Networks. Our propagator interacts with external unary cost functions (linear objective) and is able to **reformulate the problem** with an explicit lower bound in minimization. When applied at every search node of branch-and-bound method, it provides competitive results to standard constraint and integer programming approaches on difficult **Quadratic Assignment Problems** and selected **XCSP3 Competition** benchmarks.

Weighted Constraint Satisfaction Problem

Definition: graphical model, aka. Cost Function Network

- A set X of discrete variables
- A set D of finite domains
- A set F of potential functions or cost functions
- For each function f_S : $\prod_{x \in S} D_x \to \mathbb{N} or \mathbb{R} \bigcup \{\infty\}$

Objective:

Find a complete assignment x minimizing $F(x) = \sum_{f_S \in F_i} f_S(x|_S)$ where $\boldsymbol{x}$ is a complete assignment of $\boldsymbol{X}$ and $x|_S$ denotes the projection of $\boldsymbol{x}$ to variables $\boldsymbol{S}$.

NP-Hard Problem

(Cooper 2010)

Example

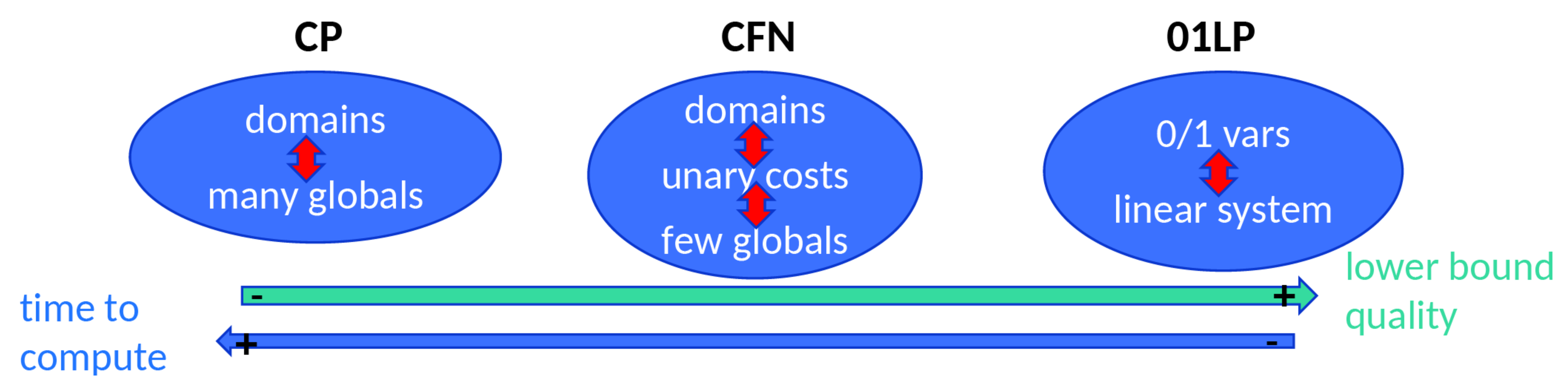
x	y	f_{xy}
a	a	3
a	b	2
b	a	0
b	b	∞

x	f_x
a	0
b	2

y	f_y
a	0
b	2

$$\min_{x,y} \quad f_x(x) + f_y(y) + f_{xy}(x,y)$$

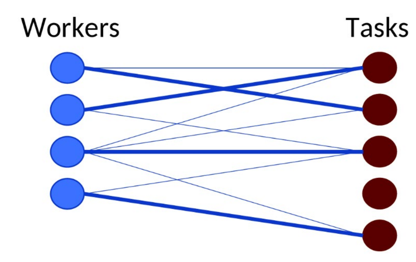
$x = b, y = a$ is the optimal assignment with cost 2.

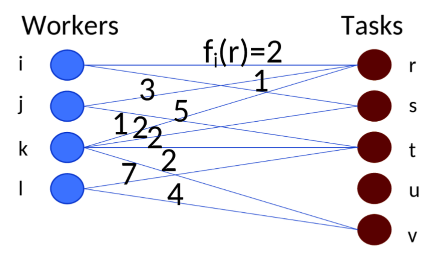


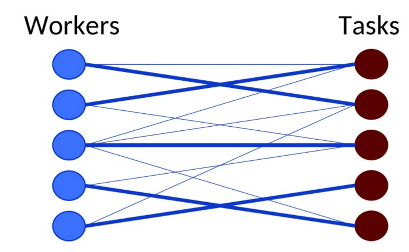
AllDifferent – non-permutation case (|X| < |D|)

AllDifferent – permutation case (|X| = |D|)

(Burkard, Dell'Amico, Martello 2012)







Constraint Programming

Google Or-tools CP-SAT solver

MinWeightAllDifferent(Workers, f, ub)

$\equiv$ AllDifferent($\{X_i, X_j, X_k, X_l\}$) $\wedge$ $\Sigma f_i(X_i) < ub$

(Régin 1994)

(Caseau, Laburthe 1997)
(Sellmann 2002)
(Claus, Cambazard 2020)

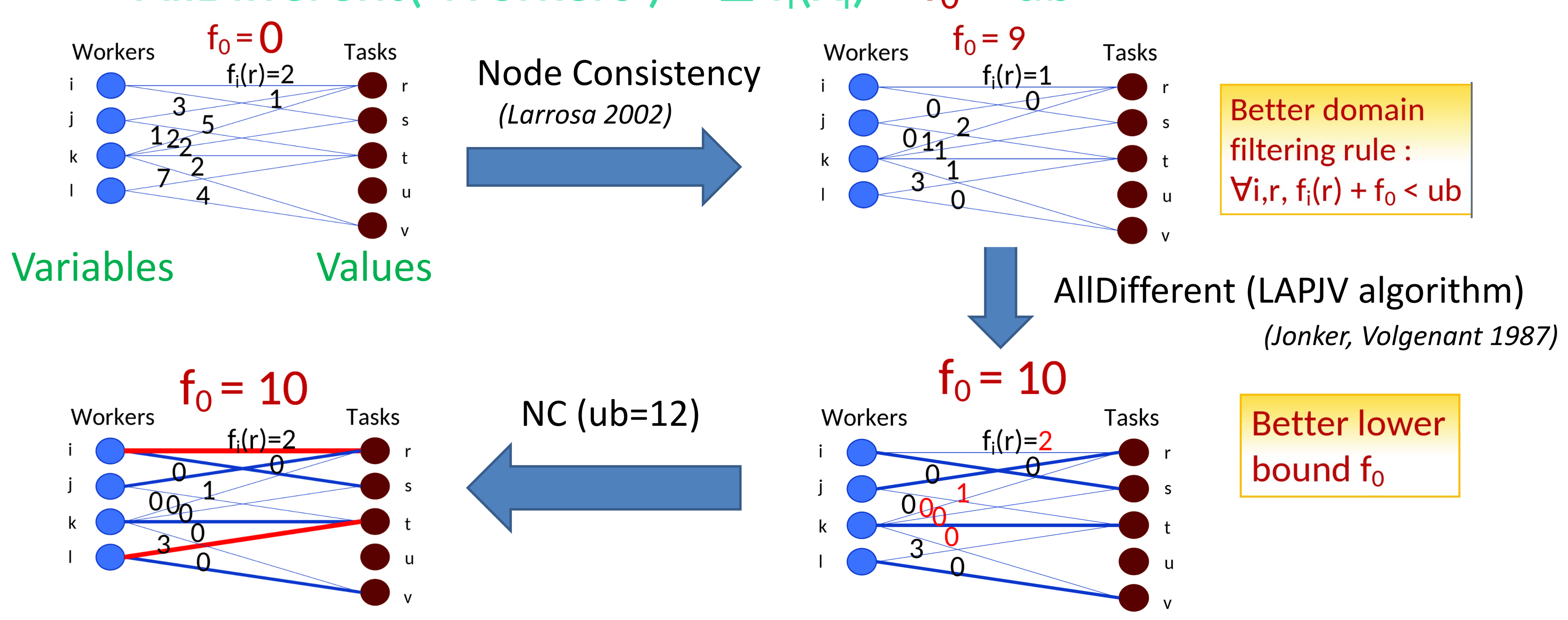
Main drawbacks

- Communication between constraints only through domains
- Incrementality issue during search (unary costs are always the same)

Cost Function Network

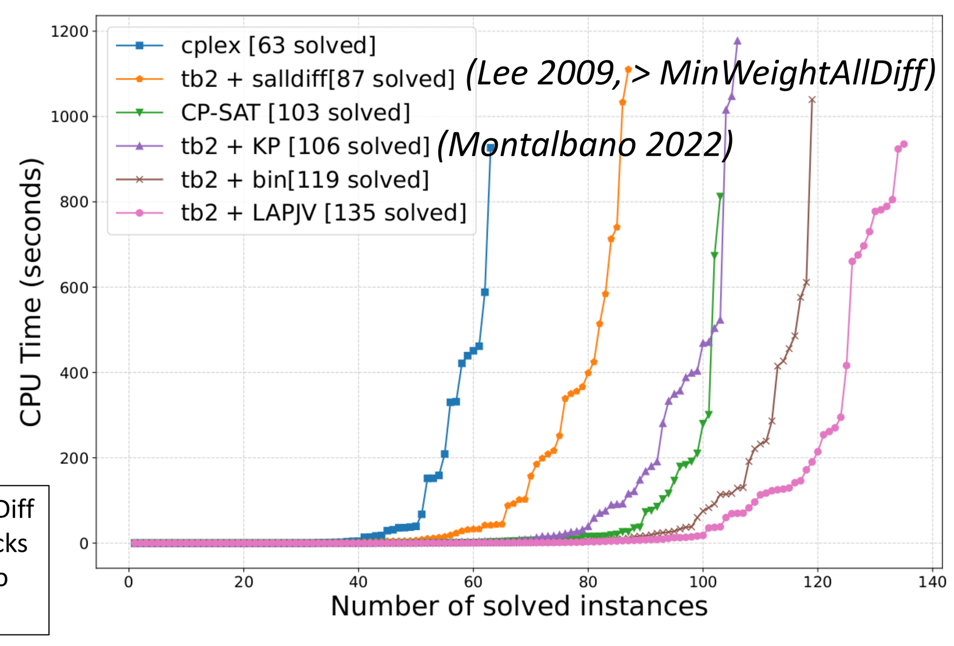
toulbar2 solver

AllDifferent(Workers) + $\Sigma f_i(X_i)$ + f_0 < ub



Computational results

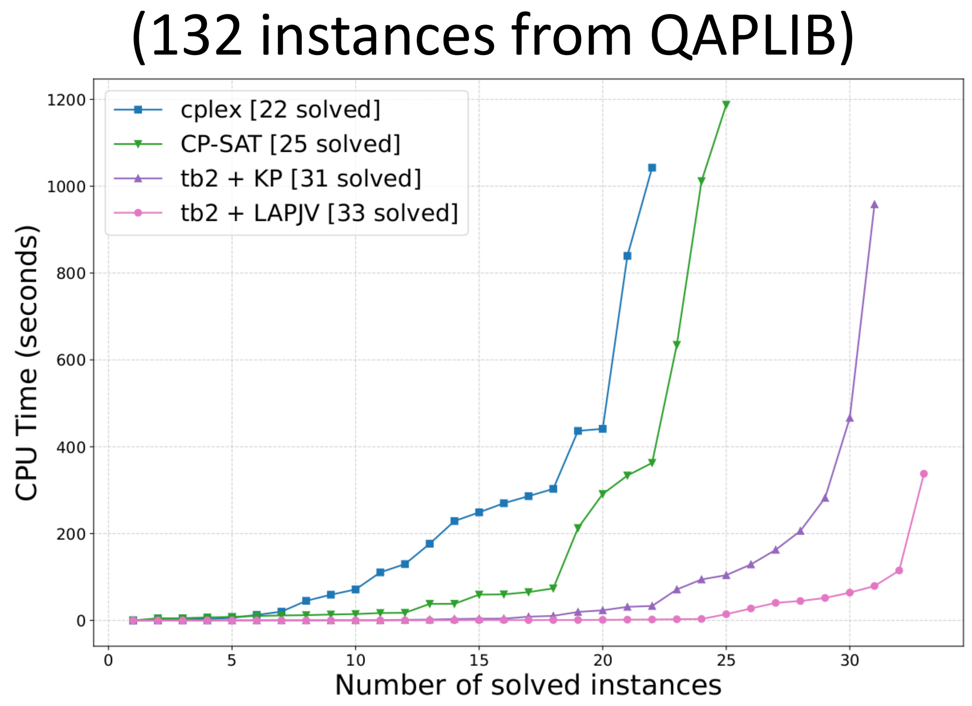
Weighted N-Queen Problem
(140 instances from n=4 to n=30)



tb2+KP: toulbar2 with AllDiff decomposed into knapsacks
tb2+bin: decomposed into binary cost functions

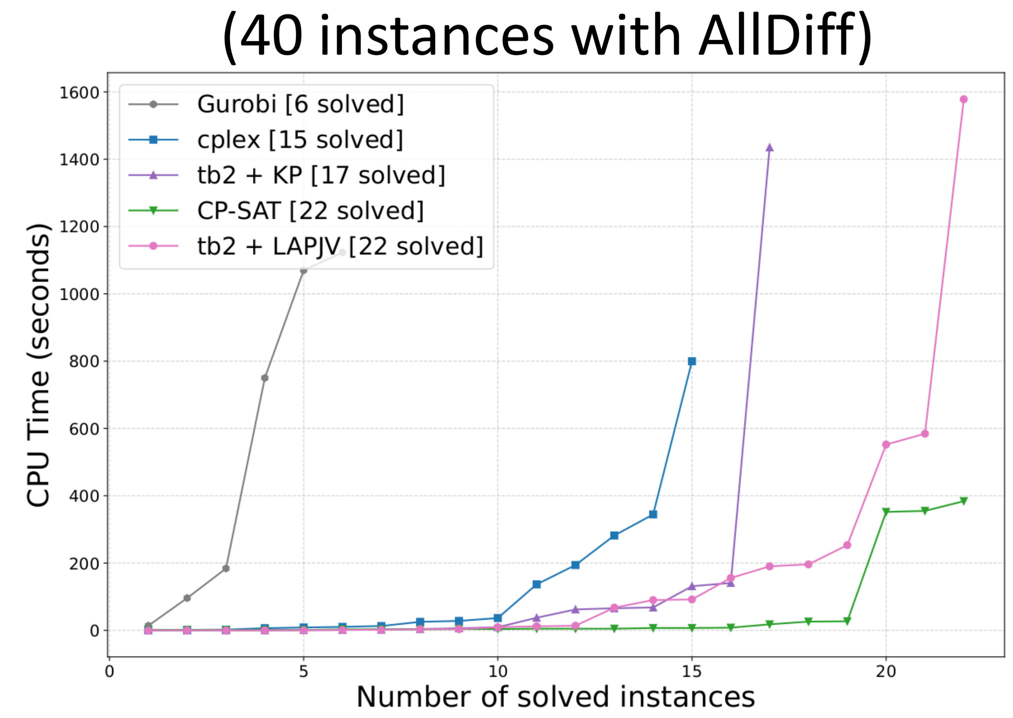
Solver N	IBM cplex	tb2 salldiff	OR-tools CP-SAT	tb2 KP	tb2 binary	tb2 LAPJV
Score	63.5	90.0	105.5	109.5	126.5	138.0

Quadratic Assignment Problem
(132 instances from QAPLIB)



Solver	cplex	CP-SAT	*tb2+at-least-one*	*tb2*+LAPJV
Score	25	43.5	57.5	**100**
gap	16% (107)	5.4%(121)	5.3 % (131)	**3.9 %(131)**

XCSP 2024 Mini COP
(40 instances with AllDiff)



Solver	gurobi	cplex	tb2+KP	CP-SAT	tb2+LAPJV
Score	6	15	17	23.5	**24.5**

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https://github.com/toulbar2/toulbar2
pip install pytoulbar2