Assignment Problems in Cost Function Networks

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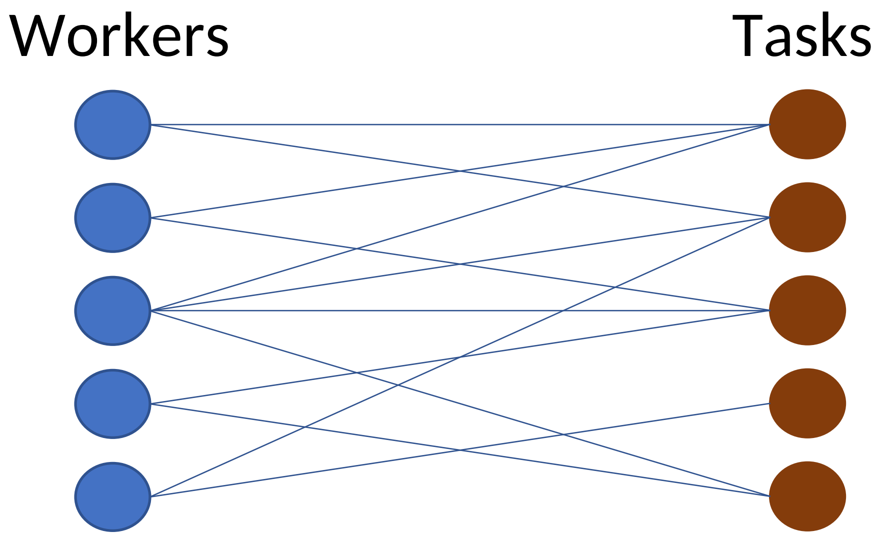
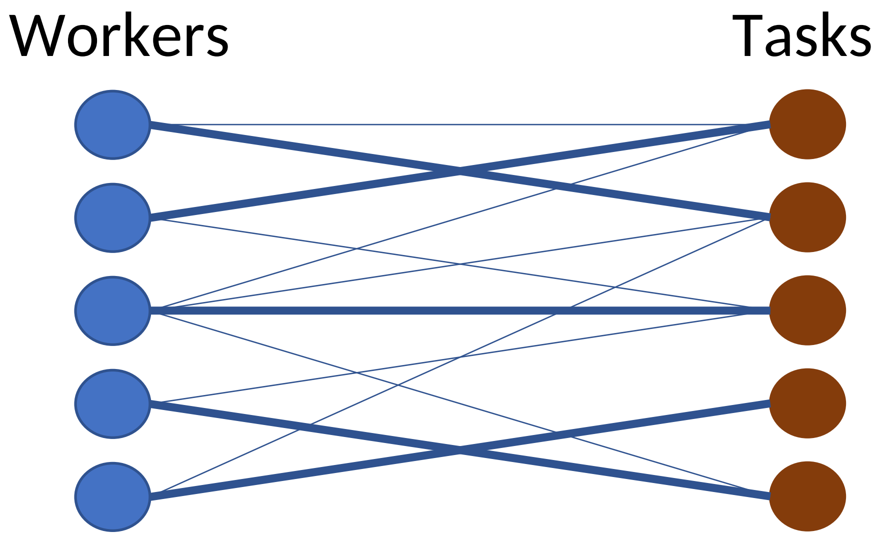
Assignment Problems

(Burkard, Dell'Amico, Martello 2012)



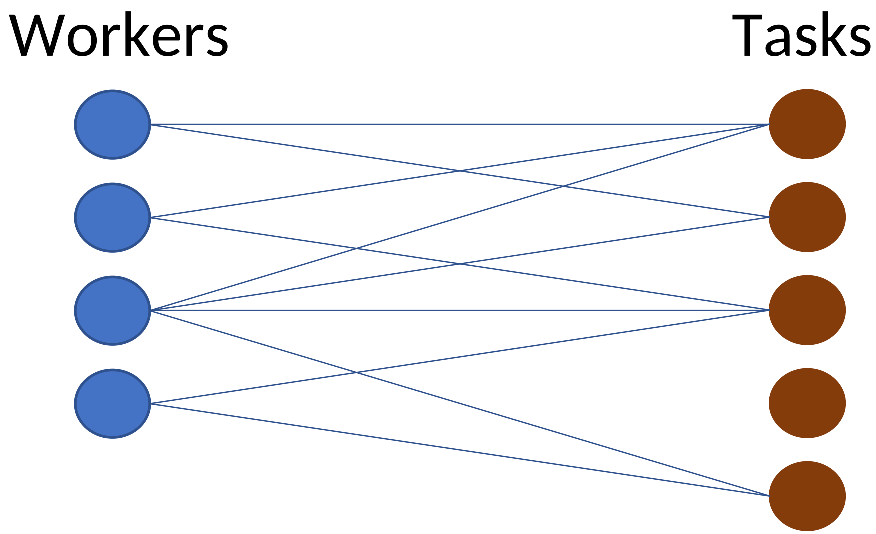
permutation case : |Workers| = |Tasks|

Assignment Problems



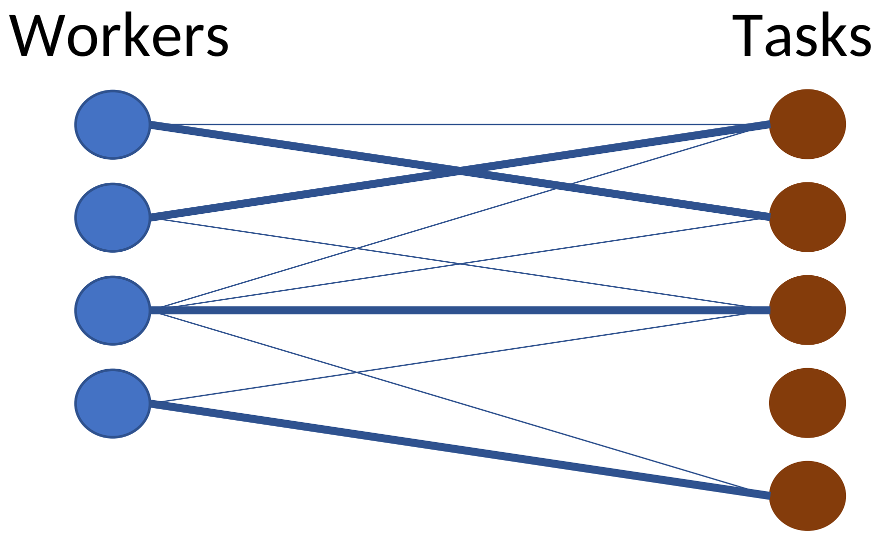
permutation case : |Workers| = |Tasks|

Assignment Problems



non-permutation case : |Workers| < |Tasks|

Assignment Problems

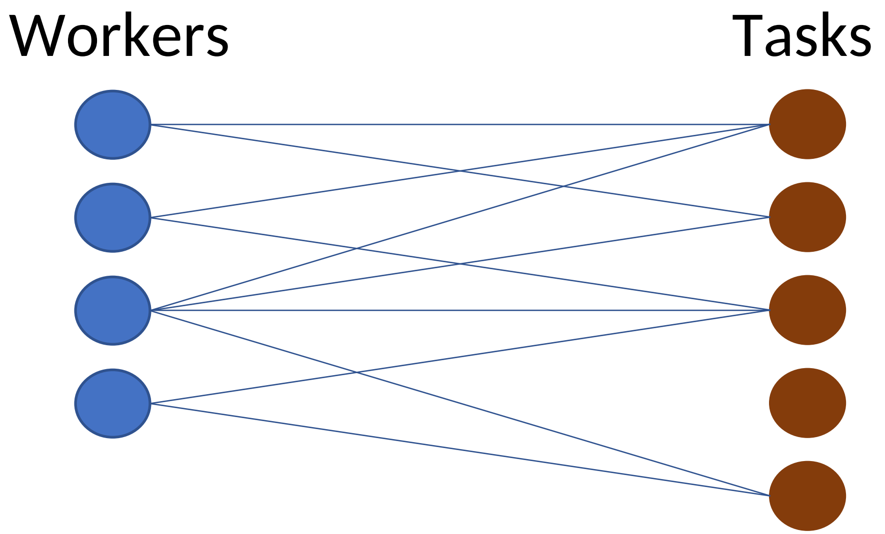


non-permutation case : |Workers| < |Tasks|

Assignment Problems .. using Constraint Programming

Google Or-tools CP-SAT solver

Variables Values

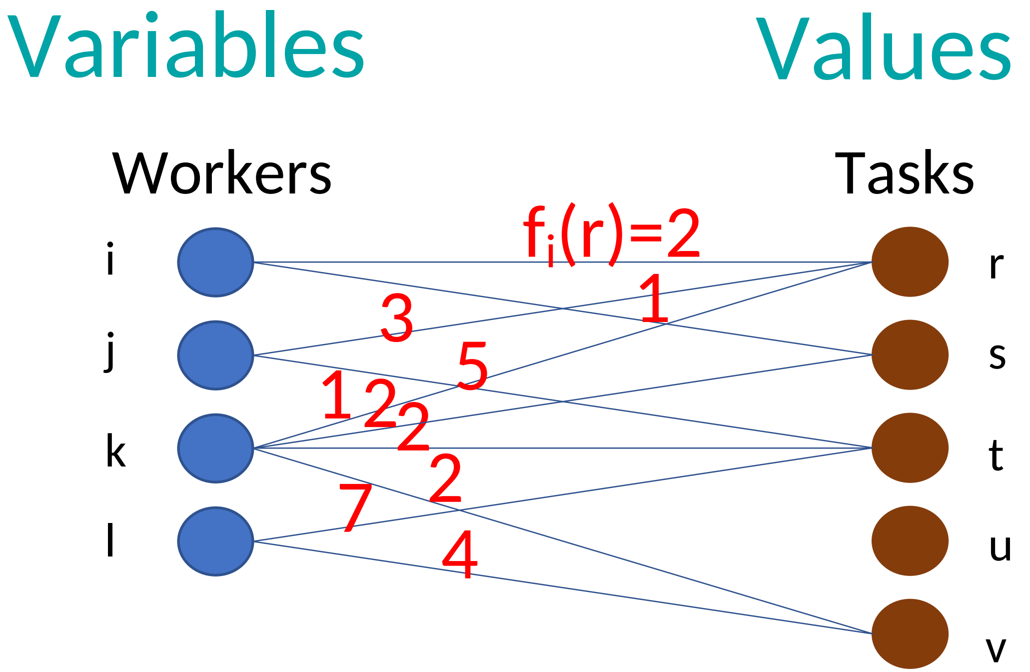


global constraint **AllDifferent(Workers)**

domain filtering in $O(n^2 \, sqrt(n))$

(Régin 1994)

Assignment Problems with unary costs in CP

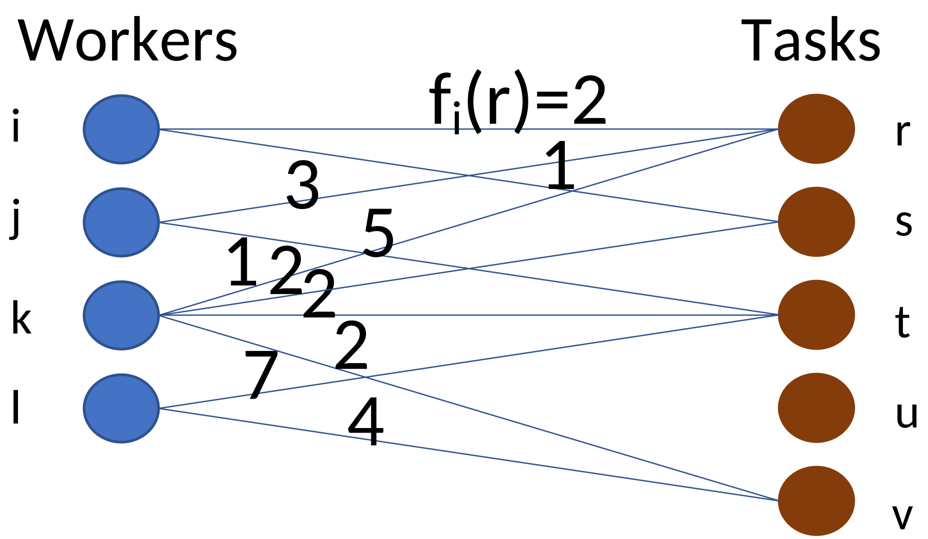


MinWeightAllDifferent(Workers, f, ub)

Assignment Problems with unary costs in CP

aka. Linear Assignment Problem

Variables Values



MinWeightAllDifferent(Workers, f, ub)

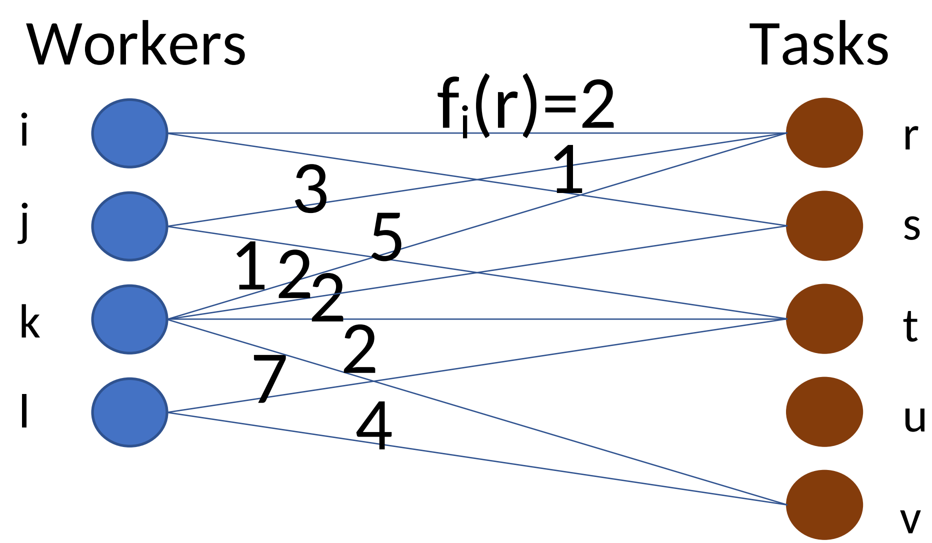
$\equiv$ AllDifferent($\{X_i, X_j, X_k, X_l\}$) $\wedge$ $\sum f_i(X_i) < ub$

domain filtering by solving an LAP in $O(n^3)$

(Caseau, Laburthe 1997)
(Sellmann 2002)

Assignment Problems with unary costs in CP

Variables Values



Main drawbacks

- **Communication between constraints only through domains**
- **Incrementality issue during search (unary costs are always the same)**

MinWeightAllDifferent(Workers, f, ub)

$\equiv$ AllDifferent($\{X_i, X_j, X_k, X_l\}$) $\wedge$ $\sum f_i(X_i) < ub$

domain filtering by solving an LAP in $O(n^3)$

(Caseau, Laburthe 1997)
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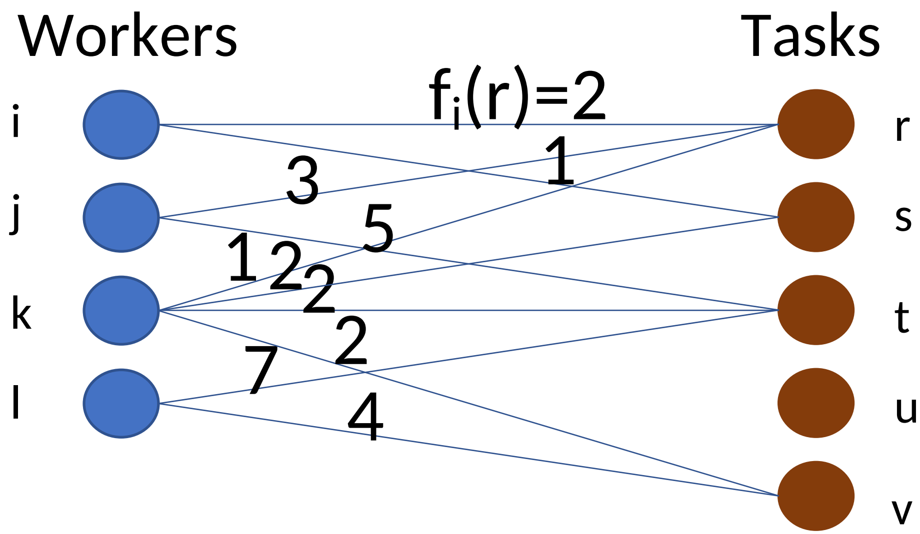
Assignment Problems in Cost Function Networks

toulbar2
solver

Variables Values



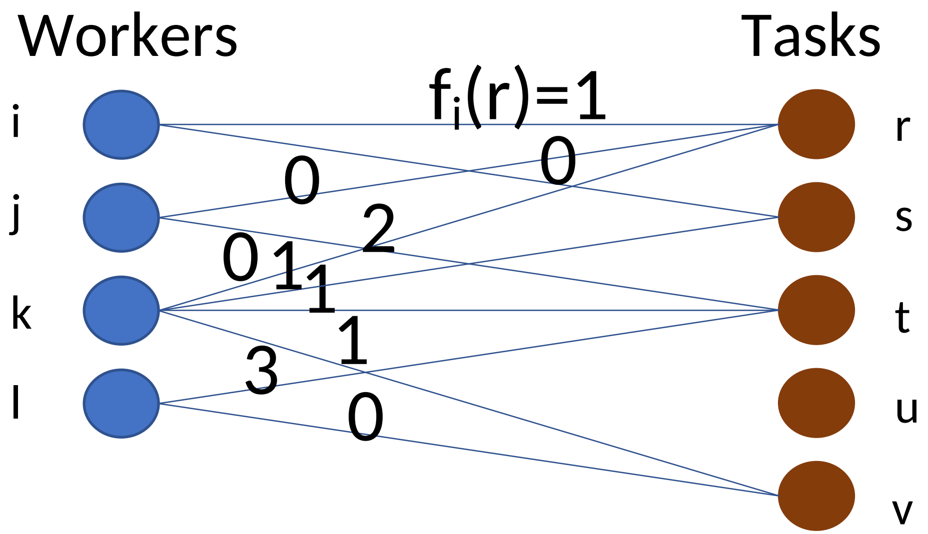
$$\text{AllDifferent(Workers)} + \sum f_i(X_i) < ub$$

global cost function

Assignment Problems in Cost Function Networks

toulbar2
solver

Variables $f_0 = 9$ Values



$$\text{AllDifferent(Workers)} + \sum f_i(X_i) + f_0 < ub$$

global cost function

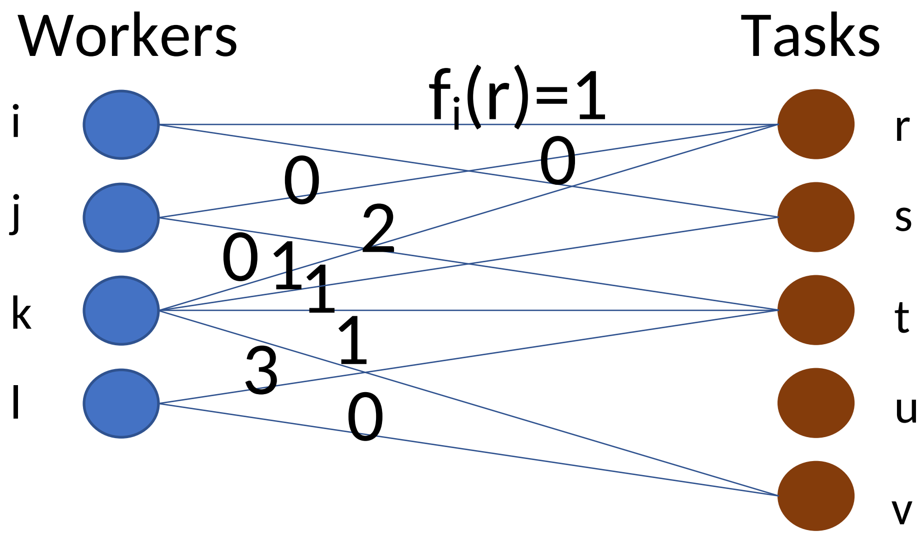
problem reformulation by Node Consistency in $O(n*d)$

(Larrosa 2002)

Assignment Problems in Cost Function Networks

toulbar2 solver

Variables $f_0 = 9$ Values



Better domain filtering rule : $\forall i,r,\ f_i(r) + f_0 < ub$

$$\text{AllDifferent(Workers)} + \sum f_i(X_i) + f_0 < ub$$

global cost function

problem reformulation by Node Consistency in $O(n*d)$

(Larrosa 2002)

Assignment Problems in Cost Function Networks

Our proposal for AllDifferent

- Permutation case (|X|=|D|)
  - We propose a reformulation with an explicit lower bound f_0 and modified unary costs (based on *reduced costs*)
- Non-permutation case (|X|<|D|)
  - We propose a reformulation with an explicit lower bound f_0, modified unary costs, and a modified AllDifferent (*delta costs*)

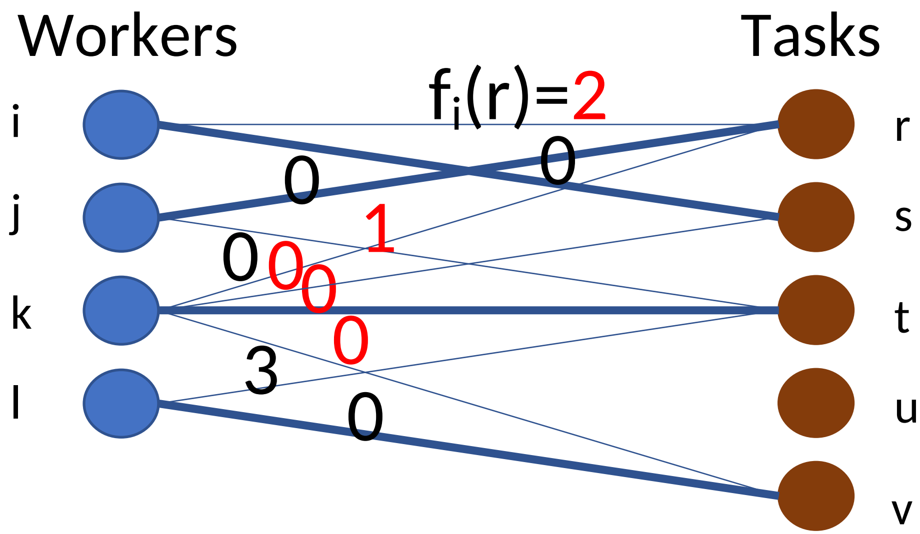
Main advantages

- **Communication between cost functions through domains and unary costs**
- **Better incrementality during search (unary costs are modified if needed)**

Assignment Problems in Cost Function Networks

toulbar2
solver

Variables $f_0 = 10$ Values



Better lower bound f_0

$$\text{AllDifferent(Workers)} + \sum f_i(X_i) + f_0 < ub$$

+ delta costs

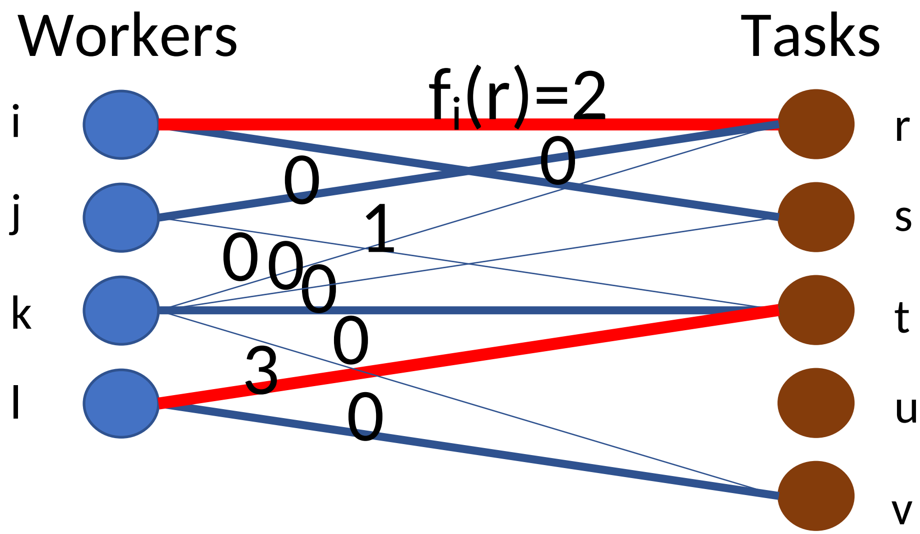
problem reformulation using LAPJV algorithm in $O(n^3)$

(Jonker, Volgenant 1987)

INRAE MIA TOULOUSE

Assignment Problems in Cost Function Networks

Variables $f_0 = 10$ Values



NC domain filtering rule :
$\forall i,r,\ f_i(r) + f_0 < ub$

AllDifferent(Workers) + $\sum f_i(X_i) + f_0 < 12$

+ delta costs

problem reformulation using LAPJV algorithm in $O(n^3)$

and domain filtering using Node Consistency

Weighted n-Queen Problem

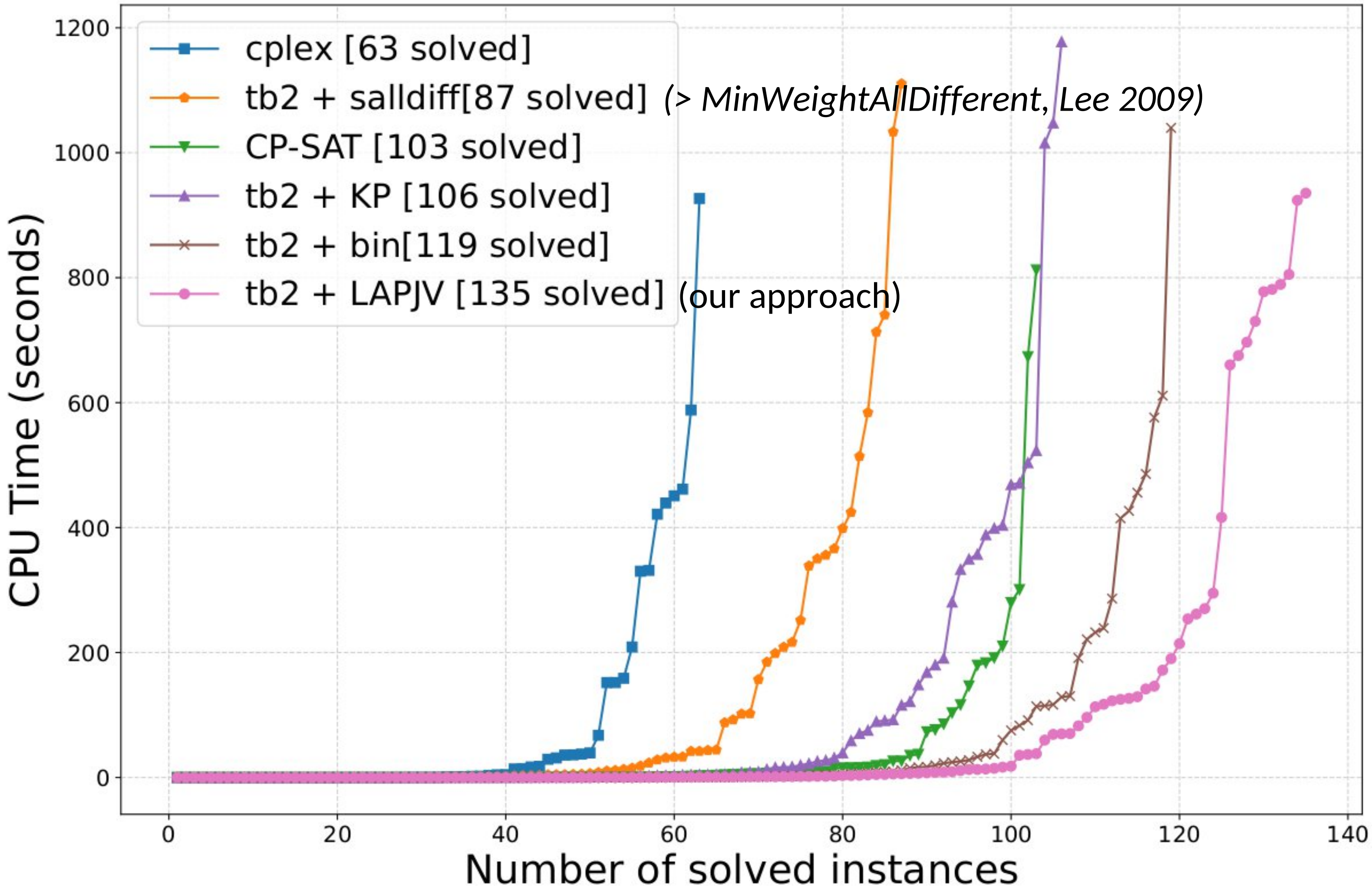
as a Cost Function Network

	X_1	X_2	X_3	X_4
1	0	4 ♕	3	0
2	3	0	1	0 ♕
3	2 ♕	1	0	1
4	5	2	2 ♕	0

- Variables
  - Queens = $\{X_1, \ldots, X_i, \ldots, X_n\}$ with domains of size n
  - Diagonal = $\{X_1-1, \ldots, X_i-i, \ldots, X_n-n\}$ with domains of size 2*n
  - AntiDiagonal = $\{X_1+1, \ldots, X_i+i, \ldots, X_n+n\}$ with domains of size 2*n

- Min AllDifferent(Queens) + — permutation case
  AllDifferent(Diagonal) + — non-permutation case
  AllDifferent(AntiDiagonal) + — non-permutation case
  $\sum f_i(X_i) + f_0$

Results on Weighted n-Queen Problem (140 instances of size n=4 to n=30)



(> MinWeightAllDifferent, Lee 2009)

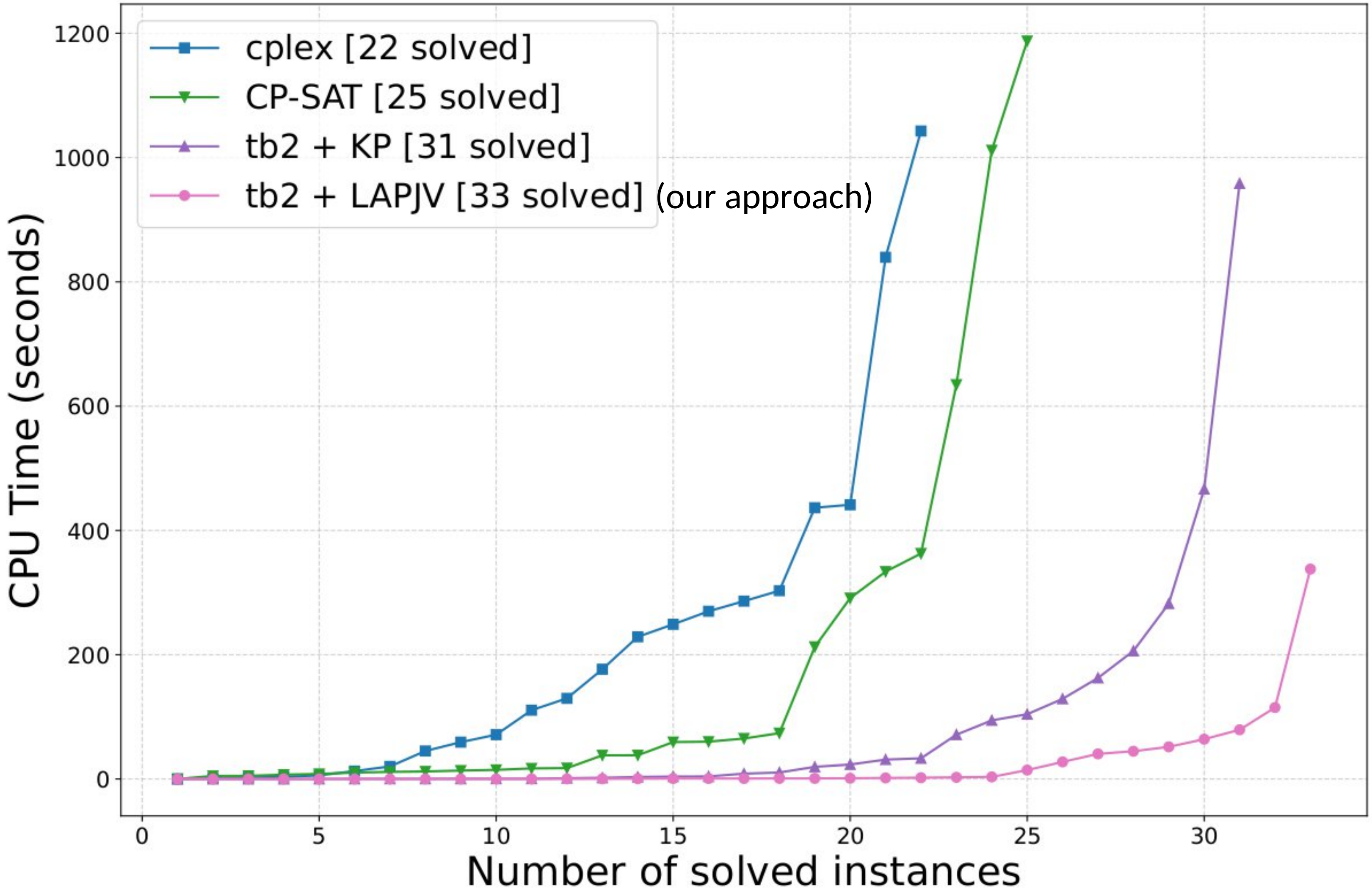
(our approach)

tb2+KP: AllDiff decomposed into knapsacks *(Montalbano 2022)*

tb2+bin: decomposed in binary cost functions

Results on Quadratic Assignment Problem (132 instances from QAPLIB)

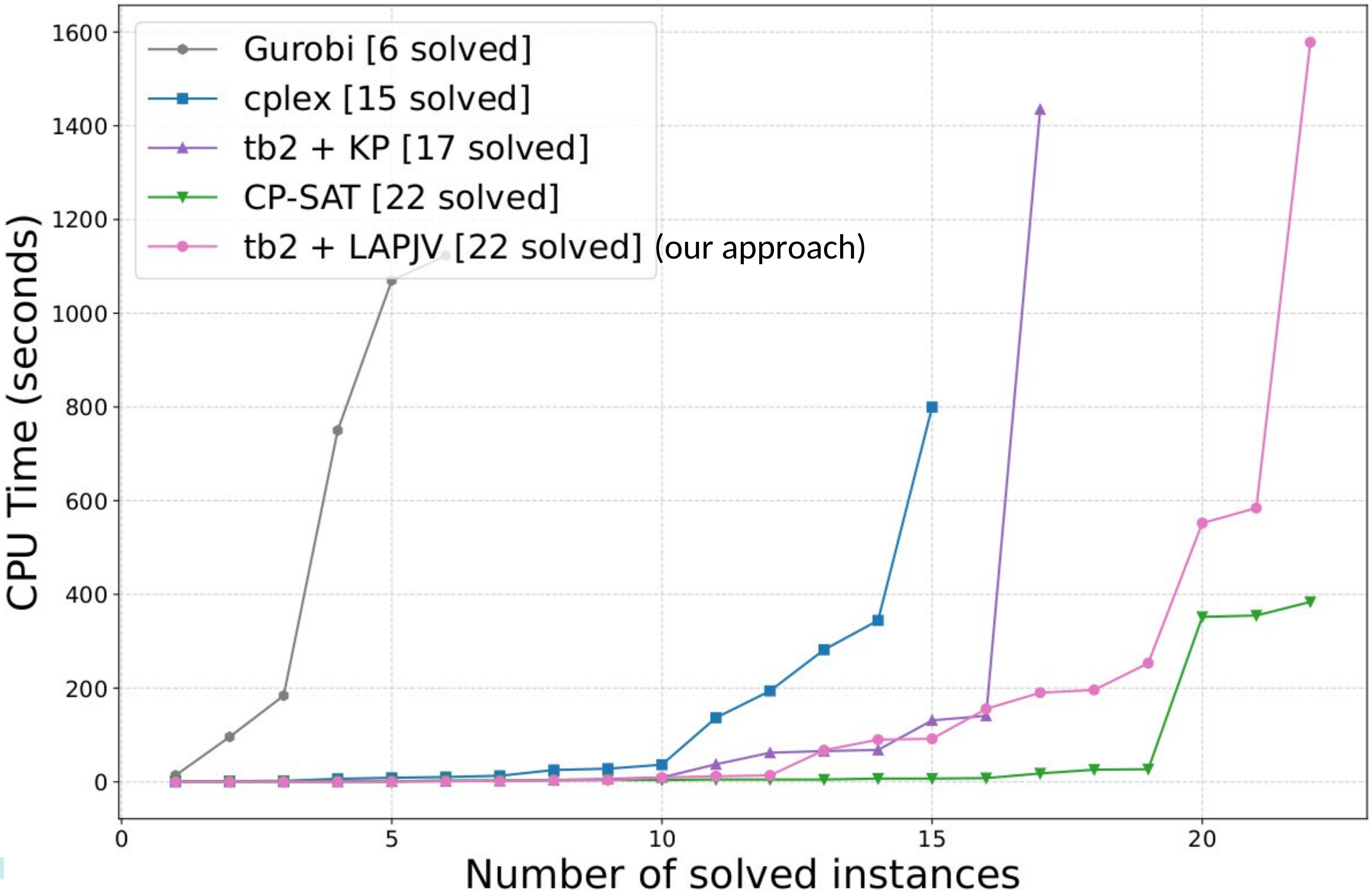
permutation case



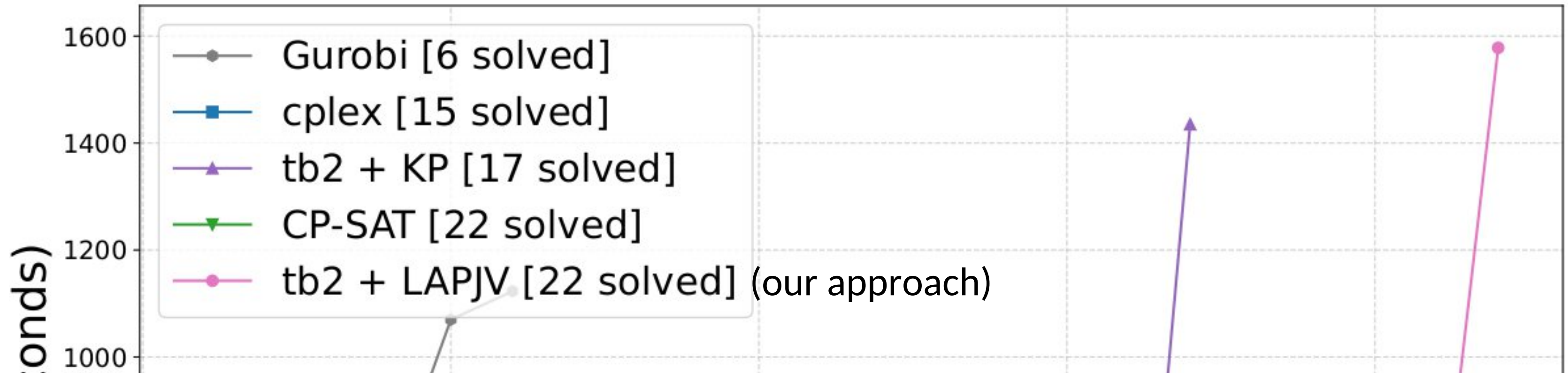
Problem reformulation by Soft Arc Consistency (EDAC) on **binary** cost functions *(Cooper 2010)*

Results on XCSP 2024 Competition MiniCOP (40 instances with AllDifferent)

permutation and non-permutation cases

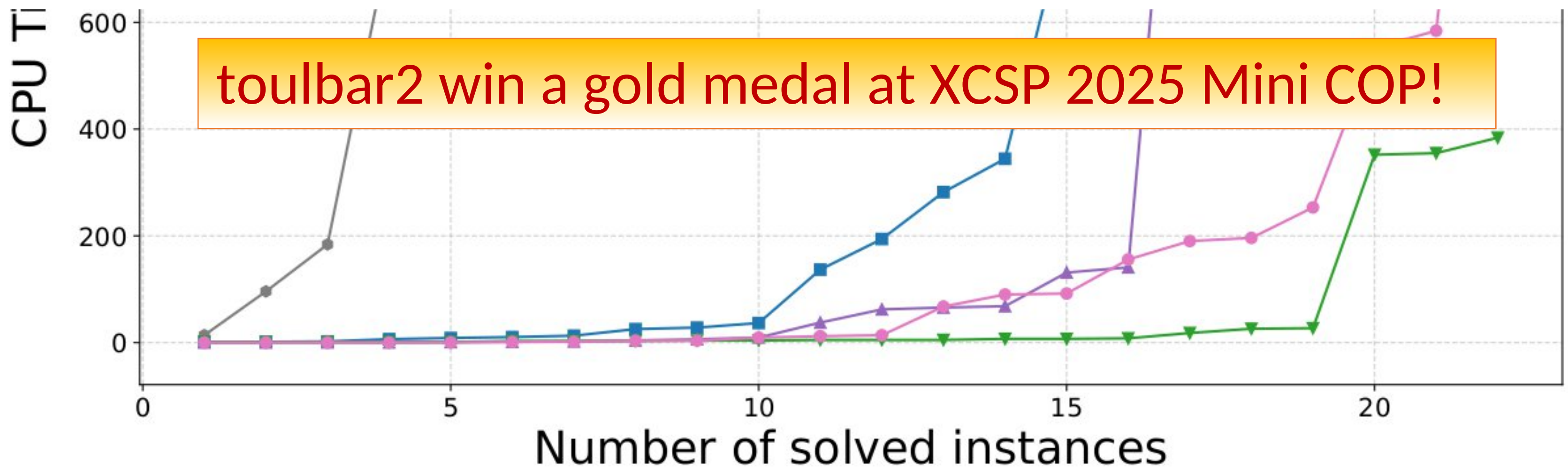


Results on XCSP 2024 Competition Mini COP

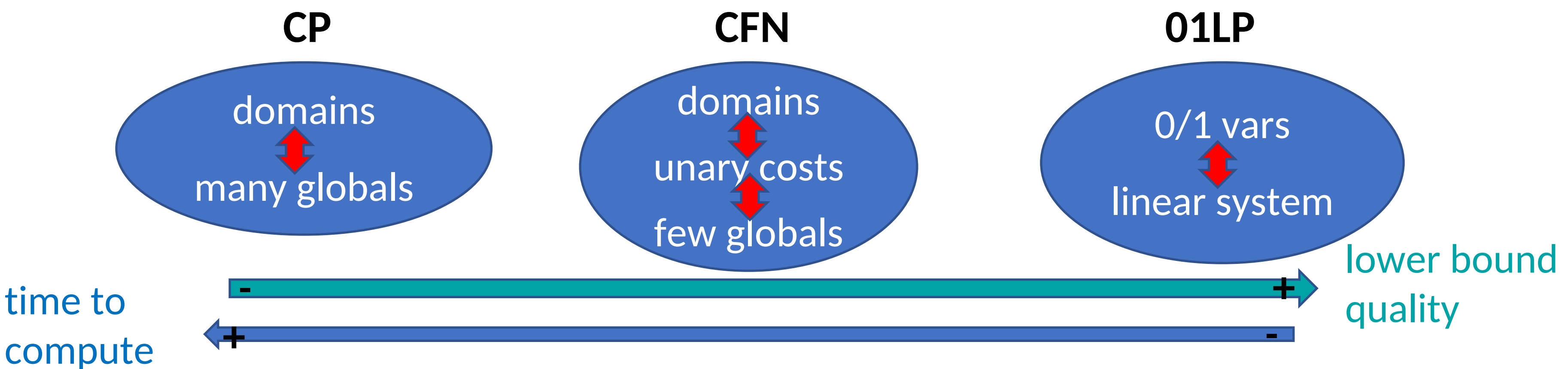


Solver	gurobi	cplex	tb2+KP	CP-SAT	tb2+LAPJV
Score	6	15	17	23.5	**24.5**

toulbar2 win a gold medal at XCSP 2025 Mini COP!



Conclusion and perspectives



- Conclusion
  - Better communication between cost functions thanks to unary cost functions
  - Reformulation in the permutation case increases the lower bound without hidding costs
- Future work

  (Sellmann 2002)(Cambazard 2020)
  - Enhance domain propagation using exact reduced costs
  - Apply the same approach to other globals (GCC,…)
  - Explore stronger reformulations (Virtual AC, singleton consistency,..)

  (Cooper 2010)

References

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Questions.

Thank you for your attention.

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Cost function network open source C++ solver
https://github.com/toulbar2/toulbar2
Python interface
pip install pytoulbar2

Many problem input formats available :
cfn, cnf, lp, opb, qpbo, uai, wbo, wcnf, wcsp, xcsp